

# Chapter 9

---

## Sinusoids and Phasors

By Eng. Emad Mahdy

WhatsApp: +201100184676

<https://www.youtube.com/channel/UC2VtseEd46wuDfmDXhfB9Ag>

<https://si-manual.com>



## Chapter Contents

- 9.1 Introduction
- 9.2 Sinusoids
- 9.3 Phasors
- 9.4 Phasor Relationships for Circuit Elements
- 9.5 Impedance and Admittance
- 9.6 †Kirchhoff's Laws in the Frequency Domain
- 9.7 Impedance Combinations

### 9.1 Introduction محتوي الشايتر

A **sinusoid** is a signal that has the form of the sine or cosine function.

## 9.2 Sinusoids

$$v(t) = V_m \cos(\omega t + \phi)$$

- $V_m \rightarrow$  Amplitude
- $\omega \rightarrow$  radian frequency
- $\phi \rightarrow$  phase
- $T \rightarrow$  period
- $f \rightarrow$  frequency

$$T = \frac{2\pi}{\omega}$$

$$f = \frac{1}{T}$$

### A periodic function

- أي periodic function لازم تحقق العلاقة دي

$$f(t) = f(t + nT)$$

#### ملاحظات:

- في كل المسائل بنشتغل فقط على  $\cos$  ولو اعطاني المسألة على شكل  $\sin$  لازم احول المسألة ل  $\cos$
- للتحويل من  $\sin$  الي  $\cos$  نطرح 90
- للتحويل من  $-\cos$  الي  $\cos$  اجمع 180

Ex:

$$v(t) = 10 \sin(5t + 70) = 10 \cos(5t + 70 - 90) = 10 \cos(5t - 20)$$

Ex:

$$v(t) = -10 \cos(5t + 20) = 10 \cos(5t + 20 + 180) = 10 \cos(5t + 200)$$

Ex:

$$v(t) = -20 \sin(5t + 40)$$

$$v(t) = -20 \sin(5t + 40) = 20 \sin(5t + 40 + 180) = 10 \sin(5t + 220)$$

$$v(t) = 10 \sin(5t + 220) = 10 \cos(5t + 220 - 90) = 10 \cos(5t + 130)$$

- للتحويل من  $-\cos$  الي  $\cos$  اجمع 180

- اطرح 90 عشان احول من  $\sin$  الي  $\cos$

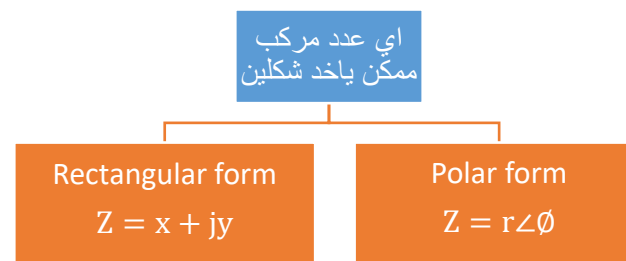
## 9.3 Phasors

A **phasor** is a complex number that represents the amplitude and phase of a sinusoid.

■ للتحويل من phasor الي sinusoids

sinusiod	phasor
$v(t) = V_m \cos(\omega t + \phi)$	$V = V_m \angle \phi$
Time domain	Frequency domain
$v(t)$ is time dependant	$V$ is time independant
All symbols are small.	All symbols are capital.

*Note: Phasor analysis applies only when frequency is constant*



Time domain representation	Phasor domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

## 9.4 Circuit Elements

	Time domain	Frequency domain
Resistor R	$v = Ri$	$V = RI$
Inductor L	$v = L \frac{di}{dt}$	$V = j\omega LI$
Capacitor C	$i = C \frac{dv}{dt}$	$V = \frac{I}{j\omega C}$

## 9.5 Impedance and Admittance

$$Z = \frac{V}{I} = R + jX$$

$$Y = \frac{I}{V} = \frac{1}{Z}$$

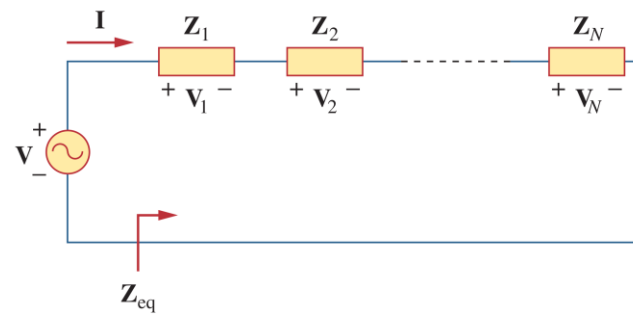
	Impedance	Admittance
$R$	$Z = R$	$Y = \frac{1}{R}$
$L$	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
$C$	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$

## 9.6 †Kirchhoff's Laws in the Frequency Domain

▪ نفس القوانين اللي كنا بنطبقها في ال time domain قبل كدا هي نفسها أقدر اطبقها في ال phasor domain

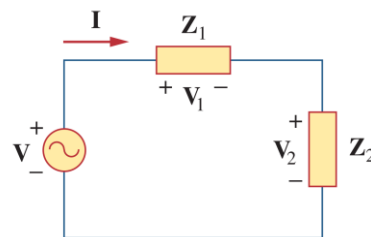
## 9.7 Impedance Combinations

### Series Impedances



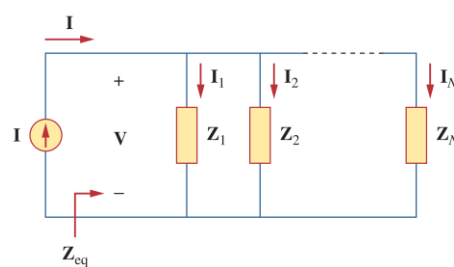
$$Z_{eq} = Z_1 + Z_2 + \dots + Z_N$$

### Voltage Division



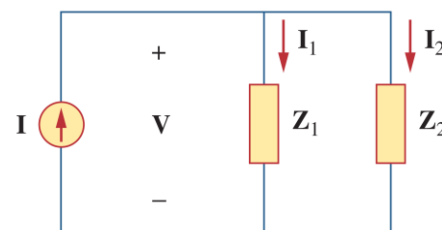
$$V_1 = \frac{Z_1}{Z_1 + Z_2} V, \quad V_2 = \frac{Z_2}{Z_1 + Z_2} V$$

### Parallel Impedances



$$Y_{eq} = Y_1 + Y_2 + \dots + Y_N$$

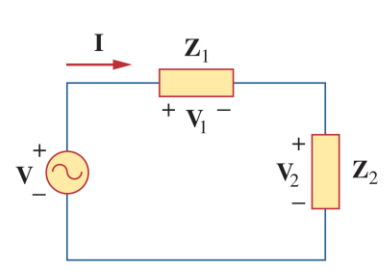
### Current Divider



$$I_1 = \frac{Z_2}{Z_1 + Z_2} I, \quad I_2 = \frac{Z_1}{Z_1 + Z_2} I$$



## Chapter Laws Summery

Chapter Laws Summery													
Sinusoids	$v(t) = V_m \cos(\omega t + \phi)$ <ul style="list-style-type: none"> <li>▪ <math>V_m \rightarrow</math> Amplitude</li> <li>▪ <math>\omega \rightarrow</math> radian frequency</li> <li>▪ <math>\phi \rightarrow</math> phase</li> <li>▪ <math>T \rightarrow</math> period</li> <li>▪ <math>f \rightarrow</math> frequency</li> </ul> $T = \frac{2\pi}{\omega}$ $f = \frac{1}{T}$												
Circuit Elements	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="width: 20%;"></th> <th style="background-color: #c00000; color: white;">Time domain</th> <th style="background-color: #c00000; color: white;">Frequency domain</th> </tr> </thead> <tbody> <tr> <td style="text-align: left;">Resistor R</td> <td><math>v = Ri</math></td> <td><math>V = RI</math></td> </tr> <tr> <td style="text-align: left;">Inductor L</td> <td><math>v = L \frac{di}{dt}</math></td> <td><math>V = j\omega LI</math></td> </tr> <tr> <td style="text-align: left;">Capacitor C</td> <td><math>i = C \frac{dv}{dt}</math></td> <td><math>V = \frac{I}{j\omega C}</math></td> </tr> </tbody> </table>		Time domain	Frequency domain	Resistor R	$v = Ri$	$V = RI$	Inductor L	$v = L \frac{di}{dt}$	$V = j\omega LI$	Capacitor C	$i = C \frac{dv}{dt}$	$V = \frac{I}{j\omega C}$
	Time domain	Frequency domain											
Resistor R	$v = Ri$	$V = RI$											
Inductor L	$v = L \frac{di}{dt}$	$V = j\omega LI$											
Capacitor C	$i = C \frac{dv}{dt}$	$V = \frac{I}{j\omega C}$											
Impedance and Admittance	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="width: 20%;"></th> <th style="background-color: #c00000; color: white;">Impedance</th> <th style="background-color: #c00000; color: white;">Admittance</th> </tr> </thead> <tbody> <tr> <td style="text-align: left;"><math>R</math></td> <td><math>Z = R</math></td> <td><math>Y = \frac{1}{R}</math></td> </tr> <tr> <td style="text-align: left;"><math>L</math></td> <td><math>Z = j\omega L</math></td> <td><math>Y = \frac{1}{j\omega L}</math></td> </tr> <tr> <td style="text-align: left;"><math>C</math></td> <td><math>Z = \frac{1}{j\omega C}</math></td> <td><math>Y = j\omega C</math></td> </tr> </tbody> </table>		Impedance	Admittance	$R$	$Z = R$	$Y = \frac{1}{R}$	$L$	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$	$C$	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$
	Impedance	Admittance											
$R$	$Z = R$	$Y = \frac{1}{R}$											
$L$	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$											
$C$	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$											
Kirchhoff's Laws in the Frequency Domain	<p style="text-align: right;">▪ نفس القوانين اللي كنا بنطبقها في ال time domain قبل كدا هي نفسها أقدر اطبقها في ال phasor domain</p>												
Impedance Combinations	<p style="color: #c00000;">Voltage Division</p> <div style="text-align: center;">  </div> $V_1 = \frac{Z_1}{Z_1 + Z_2} V, \quad V_2 = \frac{Z_2}{Z_1 + Z_2} V$ <p style="color: #c00000;">Current Divider</p>												

