Second-Order Circuits

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solution manual

Content

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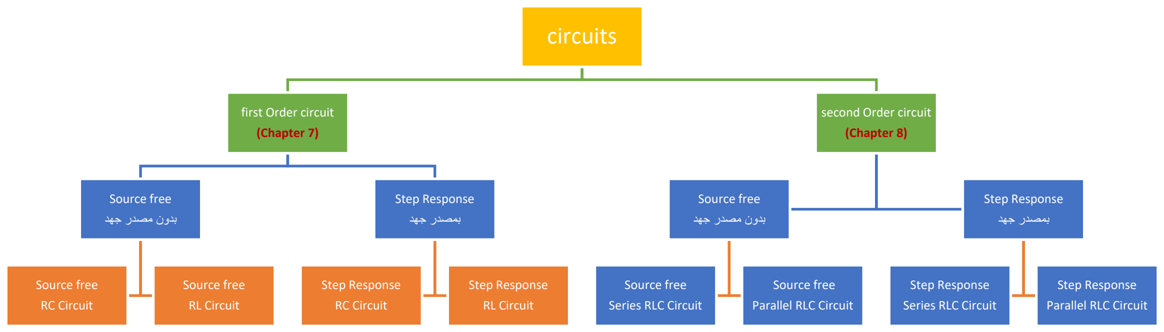
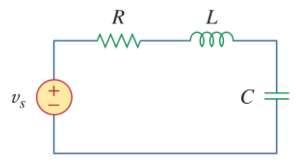
8.1 Introduction

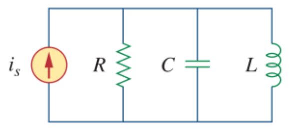
A second-order circuit is characterized by a second-order differential equation. It consists of resistors and the equivalent of two energy storage elements.



Examples of RLC circuits



(a)



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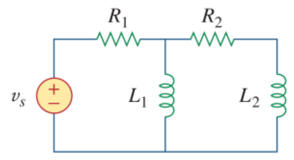
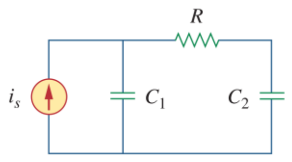


(c)



(d)

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8.2 Finding Initial and Final Values

Capacitor	Inductor
The capacitor voltage is always continuous so that: $$v_C(0^+) = v_C(0^-)$$ $$\frac{dv_c(0^+)}{dt} = \frac{i_C(0^+)}{C}$$	The inductor current is always continuous so that: $$i_L(0^+) = i_L(0^-)$$ $$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L}$$

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8.3 The Source-Free Series RLC Circuit

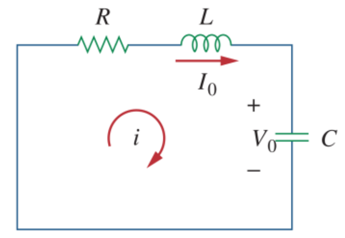
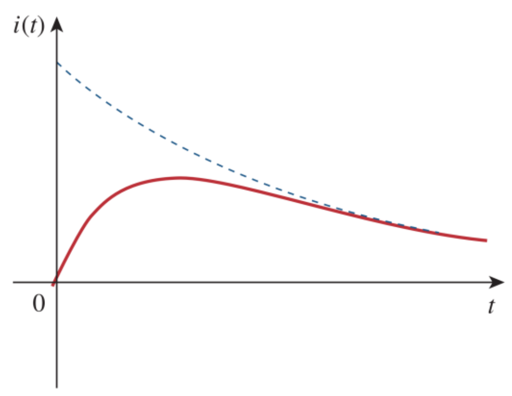
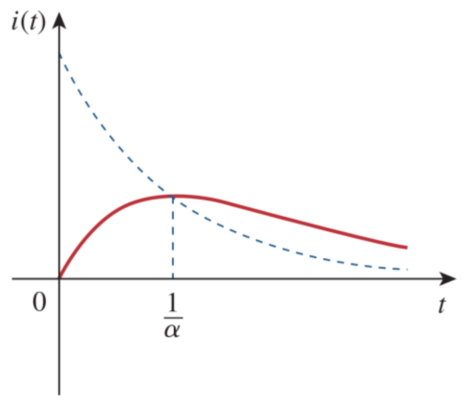
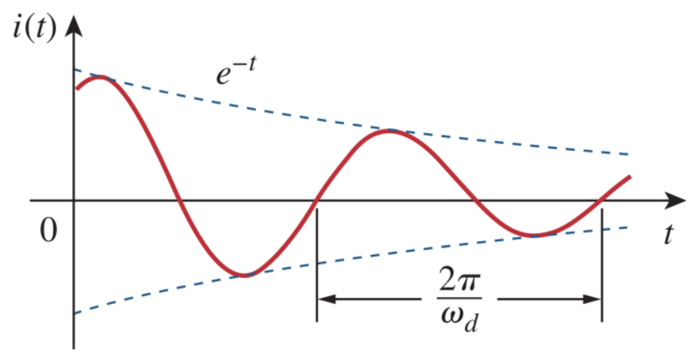


Figure 8.8
A source-free series *RLC* circuit.

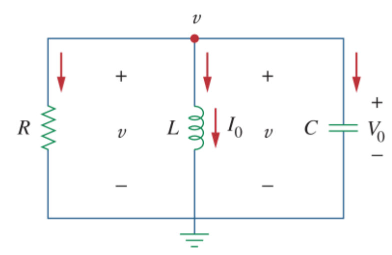
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2}, \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$$

$$\alpha = \frac{R}{2L}, \qquad \omega_o = \frac{1}{\sqrt{LC}}$$

- s_1 *and* s_2 → *natural frequencies*
- ω_o → *resonant frequency (or undamped natural frequency)*
- α → *damping factor*

Overdamped Case ($\alpha > \omega_o$)	**Critically Damped Case** ($\alpha > \omega_o$)	**Underdamped Case** ($\alpha < \omega_o$)
$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$	$i(t) = (A_2 + A_1 t)e^{-\alpha t}$	$i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$
		

8.4 The Source-Free Parallel RLC Circuit



$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2}, \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$$

$$\alpha = \frac{1}{2RC}, \qquad \omega_o = \frac{1}{\sqrt{LC}}$$

- s_1 *and* s_2 → *natural frequencies*
- ω_o → *resonant frequency (or undamped natural frequency)*
- α → *damping factor*

Overdamped Case ($\alpha > \omega_o$)	**Critically Damped Case ($\alpha > \omega_o$)**	**Underdamped Case ($\alpha < \omega_o$)**
$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$	$v(t) = (A_2 + A_1 t)e^{-\alpha t}$	$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$

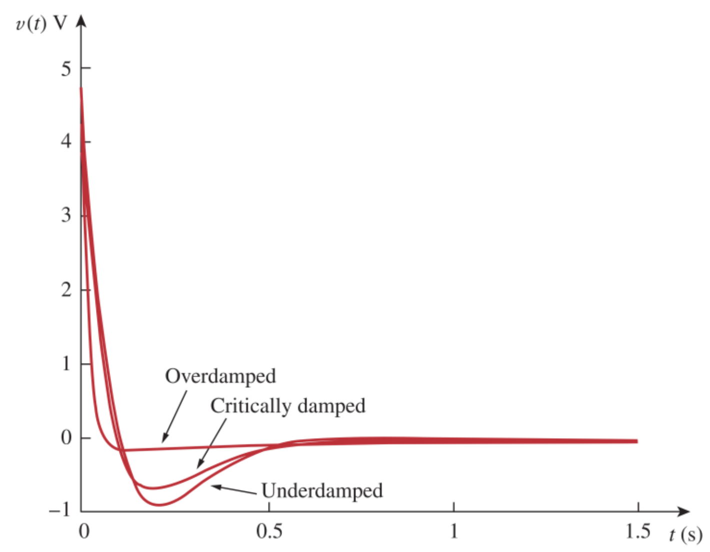


Figure 8.14
For Example 8.5: responses for three degrees of damping.

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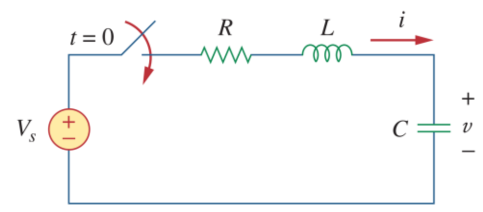
8.5 Step Response of a Series RLC Circuit



- s_1 and s_2 → natural frequencies
- ω_o → resonant frequency (or undamped natural frequency)
- α → damping factor

Overdamped	**Critically Damped**	**Underdamped**
$v(t) = V_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}$	$v(t) = V_s + (A_2 + A_1 t)e^{-\alpha t}$	$v(t) = V_s + e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$

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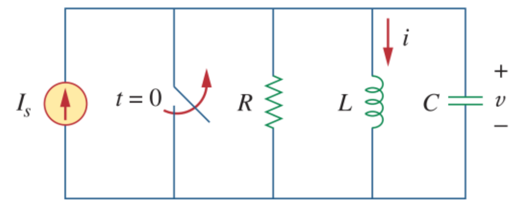
8.6 Step Response of a Parallel RLC Circuit



- s_1 and $s_2 \rightarrow$ natural frequencies
- $\omega_o \rightarrow$ resonant frequency (or undamped natural frequency)
- $\alpha \rightarrow$ damping factor

Overdamped	**Critically Damped**	**Underdamped**
$i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}$	$i(t) = I_s + (A_1 + A_2 t)e^{-\alpha t}$	$i(t) = I_s + e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$

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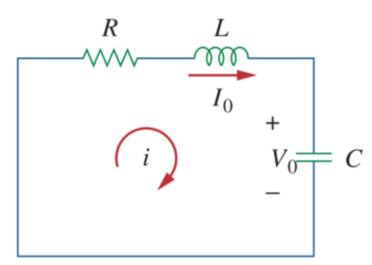
Laws Summery

Second-Order Circuits

The Source-Free

Series RLC Circuit



$$\alpha = \frac{R}{2L}$$

Overdamped Case ($\alpha > \omega_o$)	$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$
Critically Damped Case ($\alpha = \omega_o$)	$i(t) = (A_2 + A_1 t)e^{-\alpha t}$
Underdamped Case ($\alpha < \omega_o$)	$i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$

Initial condition:

$$v(0^+) = v(0^-)$$

$$i(0^+) = i(0^-)$$

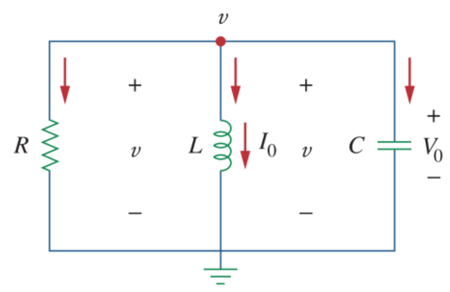
$$\frac{di(0)}{dt} = -\frac{1}{L}[Ri(0) + v(0)]$$

constants:

$$i(0)|_{t=0} \rightarrow equation\ 1$$

$$\frac{di(0)}{dt}|_{t=0} \rightarrow equation\ 2$$

Parallel RLC Circuit



$$\alpha = \frac{1}{2RC}$$

Overdamped Case ($\alpha > \omega_o$)	$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$
Critically Damped Case ($\alpha = \omega_o$)	$v(t) = (A_2 + A_1 t)e^{-\alpha t}$
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Initial condition:

$$v(0^+) = v(0^-)$$

$$i(0^+) = i(0^-)$$

$$\frac{dv(0)}{dt} = -\frac{(v(0) + Ri(0))}{RC}$$

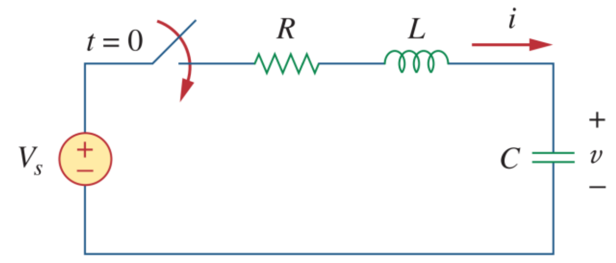
constants:

$$v(0)|_{t=0} \rightarrow equation\ 1$$

$$\frac{dv(0)}{dt}|_{t=0} \rightarrow equation\ 2$$

Step Response

Series RLC Circuit



$$\alpha = \frac{R}{2L}$$

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Initial condition:

$$v(0^+) = v(0^-)$$

$$i(0^+) = i(0^-)$$

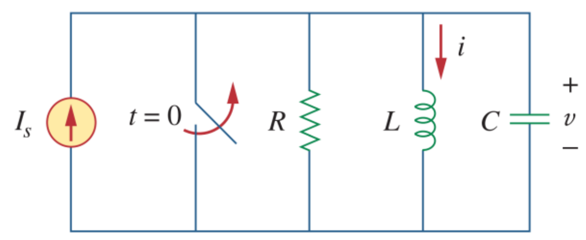
$$\frac{dv(0)}{dt} = \frac{i(0)}{C}$$

constants:

$$v(0)|_{t=0} \rightarrow equation\ 1$$

$$\frac{dv(0)}{dt}|_{t=0} \rightarrow equation\ 2$$

Parallel RLC Circuit



$$\alpha = \frac{1}{2RC}$$

Overdamped Case ($\alpha > \omega_o$)	$i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}$
Critically Damped Case ($\alpha = \omega_o$)	$i(t) = I_s + (A_1 + A_2 t)e^{-\alpha t}$
Underdamped Case ($\alpha < \omega_o$)	$i(t) = I_s + e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$

Initial condition:

$$v(0^+) = v(0^-)$$

$$i(0^+) = i(0^-)$$

$$\frac{di(0)}{dt} = \frac{v(0)}{L}$$

constants:

$$v(0)|_{t=0} \rightarrow equation\ 1$$

$$\frac{dv(0)}{dt}|_{t=0} \rightarrow equation\ 2$$

Common to all cases

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2}, \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

- s_1 and s_2 → natural frequencies
- ω_o → resonant frequency (or undamped natural frequency)
- α → damping factor

Note: Unless otherwise stated in this chapter, *v* denotes capacitor voltage, while *i* is the inductor current

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